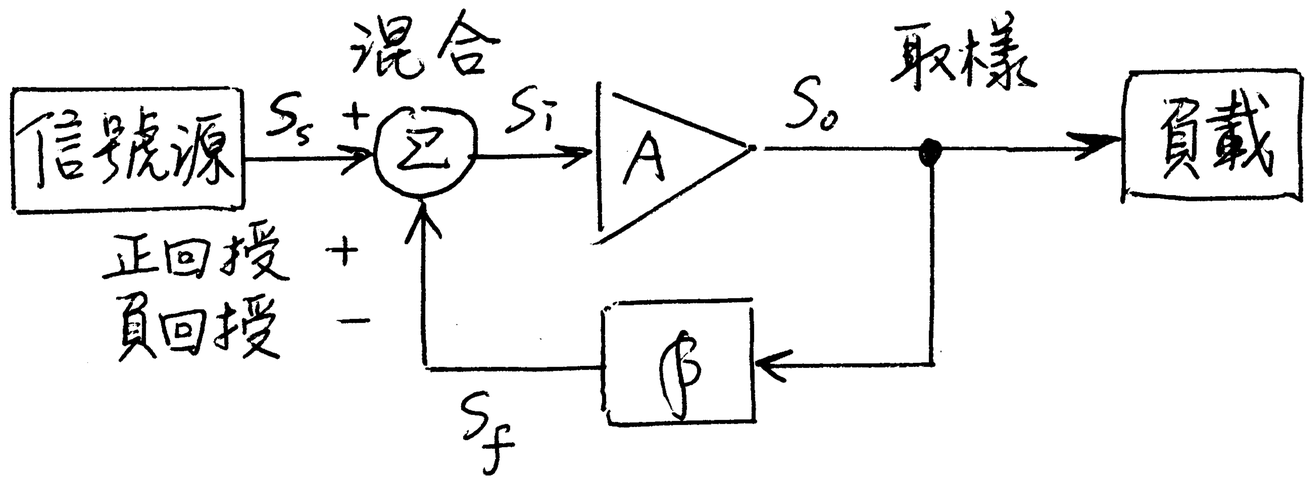
▲ 回授：從輸出信號取出一部分傳送到輸入端，使其成為輸入信號的一部分。

1. 負回授：輸入信號會被部分的輸出信號所減小，適合用於穩定放大器。

2. 正回授：部分的輸出信號會加入輸入信號，會導致不穩定或振盪。

▲ 回授結構圖



1. A：基本放大器

   β：回授網路（頻率選擇網路）

2. S_s、S_i、S_o、S_f 可以是電壓或電流

3. $S_o = A S_i$

   $S_f = \beta S_o$

4. 正回授 $S_i = S_s + S_f$

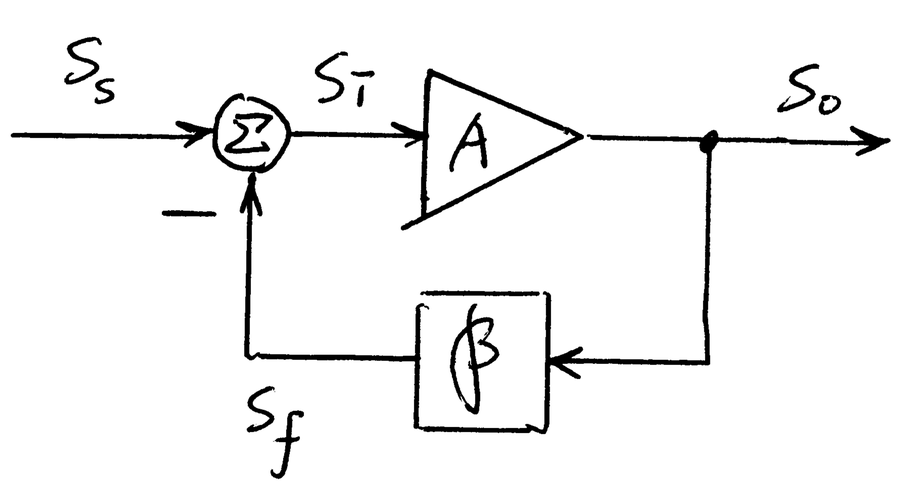
   負回授 $S_i = S_s - S_f$

▲ 閉迴路增益

正回授 $A_f = \dfrac{S_o}{S_s} = \dfrac{S_o}{S_i - S_f} = \dfrac{S_o}{\frac{S_o}{A} - \beta S_o} = \dfrac{A}{1 - \beta A}$

負回授 $A_f = \dfrac{S_o}{S_s} = \dfrac{S_o}{S_i + S_f} = \dfrac{S_o}{\frac{S_o}{A} + \beta S_o} = \dfrac{A}{1 + \beta A}$

▲ 以負回授結構圖為基礎，



$$\begin{cases} S_o = A S_i \\ S_f = \beta S_o \\ S_i = S_s - S_f \end{cases}$$

則

$$A_f = \frac{S_o}{S_s} = \frac{A}{1+\beta A} \triangleq \frac{A}{1+L}$$

其中 $L \triangleq \beta A = \frac{S_f}{S_i}$ 稱之為迴路增益 (loop gain)

(1) 正回授：$L = \beta A < 0$

(2) 負回授：$L = \beta A > 0$

▲ 振盪器：在某一特定頻率下，對於零輸入信號
具有一有限的輸出。

令輸入 $S_s = 0$，則 $\underline{S_i = -S_f}$，$\underline{S_f = \beta S_o}$，$\underline{S_o = A S_i}$

$$\Rightarrow \quad S_o = A S_i = A(-S_f) = A(-\beta S_o)$$

$$\Rightarrow \quad (1+\beta A) S_o = 0$$

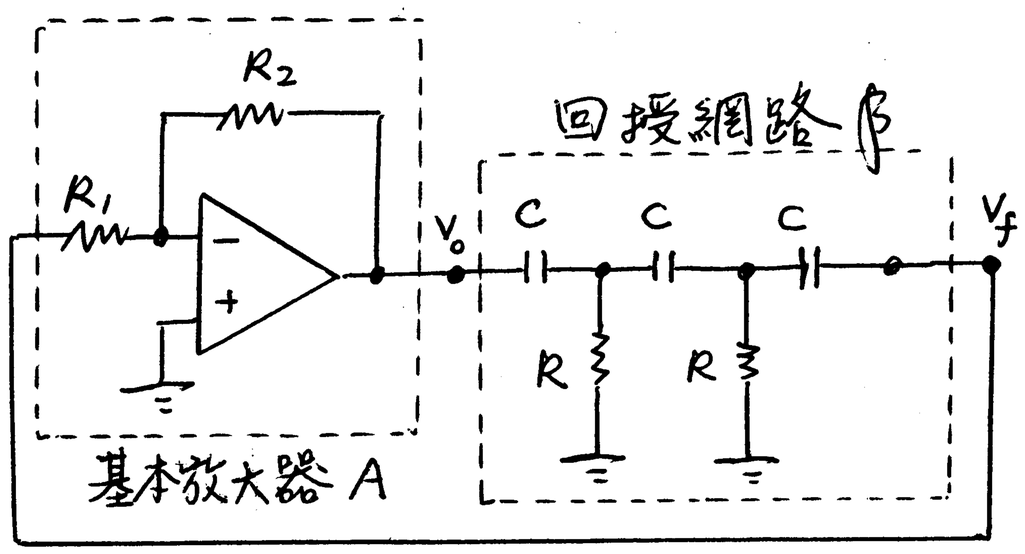
$\because S_o \neq 0 \quad \therefore \boxed{\beta A = -1}$ 為振盪器的振盪條件

▲ 巴克豪生準則：即正弦波振盪器的振盪條件。

① $\beta A = -1$，代表系統為正回授。

② $\beta A = -1 = 1 \angle 180^\circ$，代表 $|\beta A| = 1$，且 $\underline{\angle \beta A} = 180^\circ$。

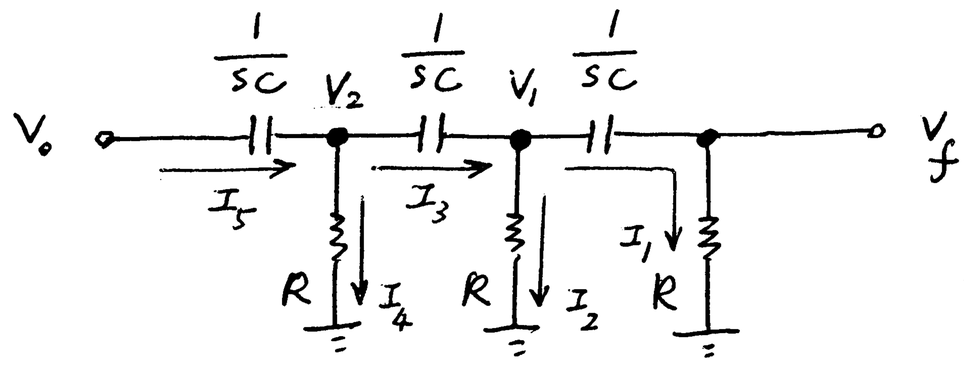
RC 相移振盪器



$$A = -\frac{R_2}{R_1}$$

$$\beta = \frac{V_f}{V_o}$$

設 $R_1 = R$



$$I_1 = \frac{V_f}{R}$$

$$V_1 = I_1\left(\frac{1}{sC} + R\right)$$

$$I_2 = \frac{V_1}{R} = I_1\left(\frac{1}{sRC} + 1\right)$$

$$I_3 = I_1 + I_2 = I_1\left(\frac{1}{sRC} + 2\right)$$

$$V_2 = I_3\frac{1}{sC} + V_1 = I_1\left(\frac{1}{s^2R^2C^2} + \frac{2}{sC}\right) + I_1\left(\frac{1}{sC} + R\right) = I_1\left(\frac{1}{s^2R^2C^2} + \frac{3}{sC} + R\right)$$

$$I_4 = \frac{V_2}{R} = I_1\left(\frac{1}{s^2R^2C^2} + \frac{3}{sRC} + 1\right)$$

$$I_5 = I_3 + I_4 = I_1\left(\frac{1}{sRC} + 2\right) + I_1\left(\frac{1}{s^2R^2C^2} + \frac{3}{sRC} + 1\right) = I_1\left(\frac{1}{s^2R^2C^2} + \frac{4}{sRC} + 3\right)$$

$$V_o = I_5\frac{1}{sC} + V_2 = I_1\left(\frac{1}{s^3R^2C^3} + \frac{4}{s^2RC^2} + \frac{3}{sC}\right) + I_1\left(\frac{1}{s^2RC^2} + \frac{3}{sC} + R\right) = I_1\left(\frac{1}{s^3R^2C^3} + \frac{5}{s^2RC^2} + \frac{6}{sC} + R\right)$$

$$\beta = \frac{V_f}{V_o} = \frac{V_f}{I_1}\frac{I_1}{V_o} = R\cdot\frac{1}{\frac{1}{s^3R^2C^3} + \frac{5}{s^2RC^2} + \frac{6}{sC} + R} = \frac{s^3R^3C^3}{s^3R^3C^3 + 6s^2R^2C^2 + 5sRC + 1}$$

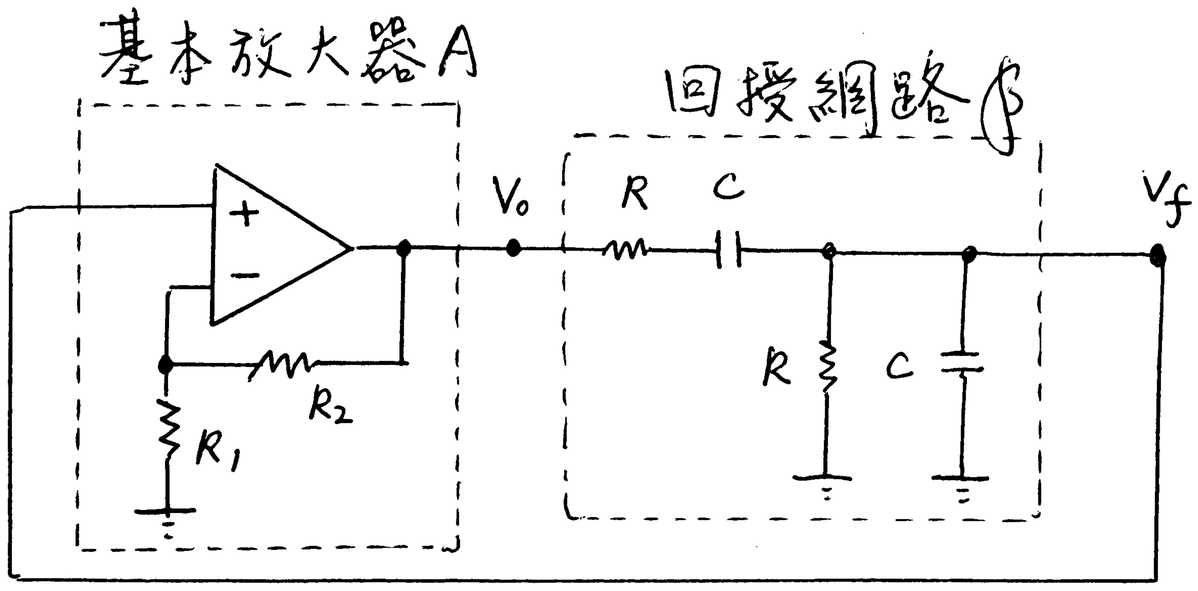
$$\beta(j\omega) = \frac{-j\omega^3R^3C^3}{(1 - 6\omega^2R^2C^2) + j(5\omega RC - \omega^3R^3C^3)}$$

當 $\omega^2R^2C^2 = \frac{1}{6}$ 時，$\beta(j\omega) = \frac{-j\frac{1}{6}\omega RC}{j(5\omega RC - \frac{1}{6}\omega RC)} = -\frac{1}{29}$，虛部為 0，

此時頻率 $\omega = \frac{1}{\sqrt{6}RC}$ 稱為振盪頻率，即 $f = \frac{1}{2\pi\sqrt{6}RC}$

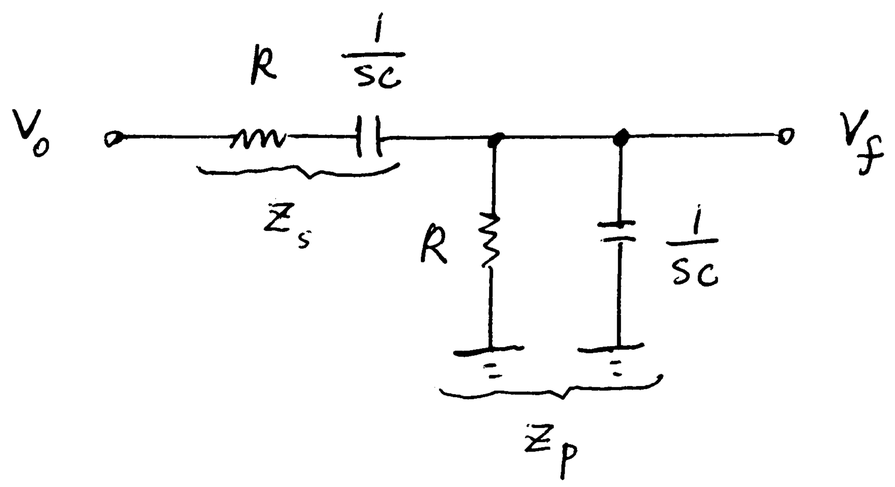
$|\beta A| = \left|(-\frac{1}{29})\times(-\frac{R_2}{R_1})\right| \geq 1$，$\frac{R_2}{R_1} \geq 29$ 為振盪條件。

Wien-bridge 振盪器



$$A = 1 + \frac{R_2}{R_1}$$

$$\beta = \frac{V_f}{V_o}$$



$$Z_s = R + \frac{1}{sC} = \frac{1+sRC}{sC}$$

$$Z_p = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{sC}}} = \frac{R}{1+sRC}$$

$$\beta = \frac{V_f}{V_o} = \frac{Z_p}{Z_p + Z_s} = \frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC} + \frac{1+sRC}{sC}} = \frac{sRC}{sRC + (1+sRC)^2} = \frac{sRC}{s^2R^2C^2 + 3sRC + 1}$$

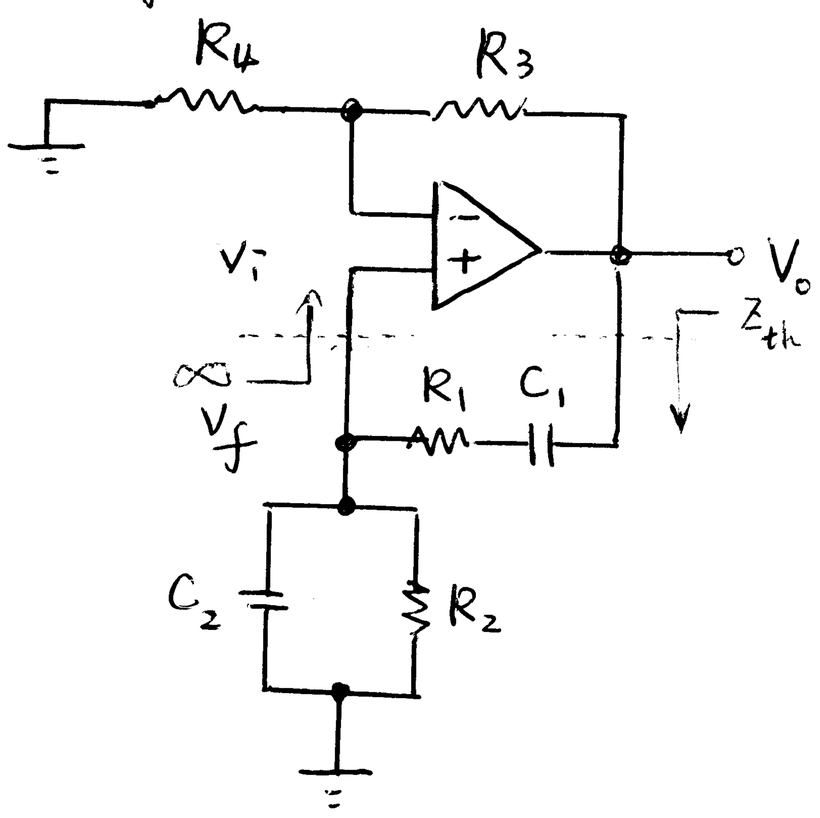
$$\beta(j\omega) = \frac{j\omega RC}{(1-\omega^2R^2C^2) + j3\omega RC}$$

當 $\omega^2R^2C^2 = 1$ 時，$\beta(j\omega) = \frac{1}{3}$，虛部為 0，

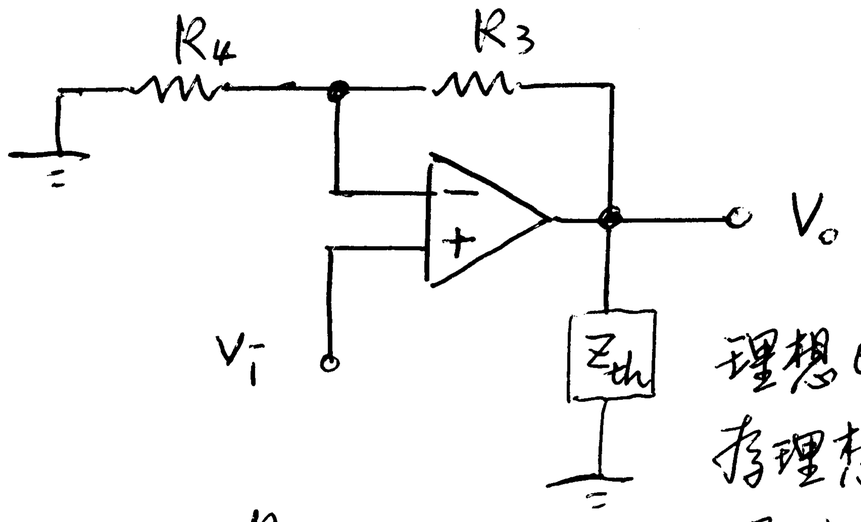
此時頻率 $\omega = \frac{1}{RC}$ 稱為振盪頻率，即 $f = \frac{1}{2\pi RC}$

$\left|\beta A\right| = \left|\frac{1}{3} \times \left(1 + \frac{R_2}{R_1}\right)\right| \geq 1$，$\frac{R_2}{R_1} \geq 2$ 為振盪條件。

Wien-bridge Oscillator



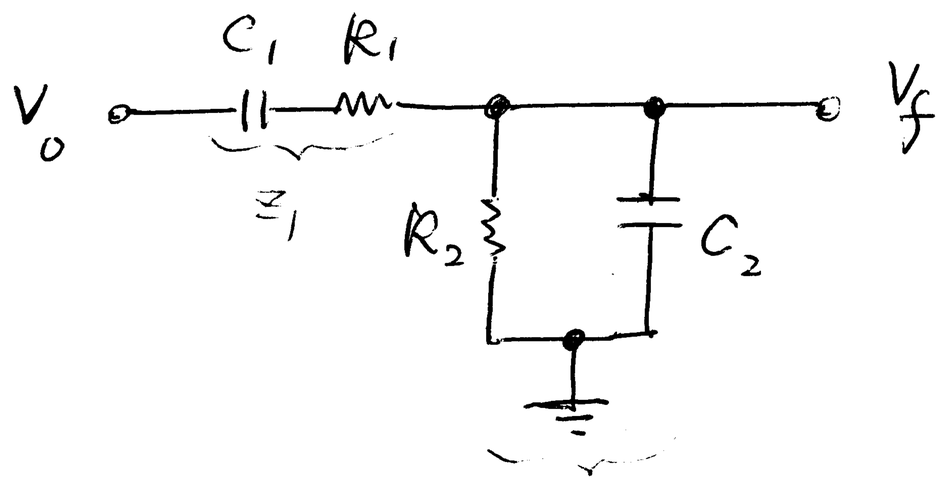
A電路



理想OPA輸出端為理想電壓源，所提供的輸出電壓与負載無関。

$$A = 1 + \frac{R_3}{R_4}$$

β電路



$$\beta = \frac{V_f}{V_o}$$

$$Z_1 = R_1 + \frac{1}{sC_1} = \frac{1+sR_1C_1}{sC_1}$$

$$Z_2 = \frac{1}{\frac{1}{R_2}+sC_2} = \frac{R_2}{1+sR_2C_2}$$

$$\beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{R_2}{1+sR_2C_2}}{\frac{1+sR_1C_1}{sC_1}+\frac{R_2}{1+sR_2C_2}} = \frac{sR_2C_1}{1+sR_1C_1+sR_2C_2+sR_2C_1+s^2R_1R_2C_1C_2}$$

$$\beta(j\omega) = \frac{j\omega R_2C_1}{j\omega(R_1C_1+R_2C_2+R_2C_1)+(1-\omega^2R_1R_2C_1C_2)}$$

當 $\omega^2R_1R_2C_1C_2 = 1$ 時，$\beta = \frac{R_2C_1}{R_1C_1+R_2C_2+R_2C_1}$ 為實數(相角為零)

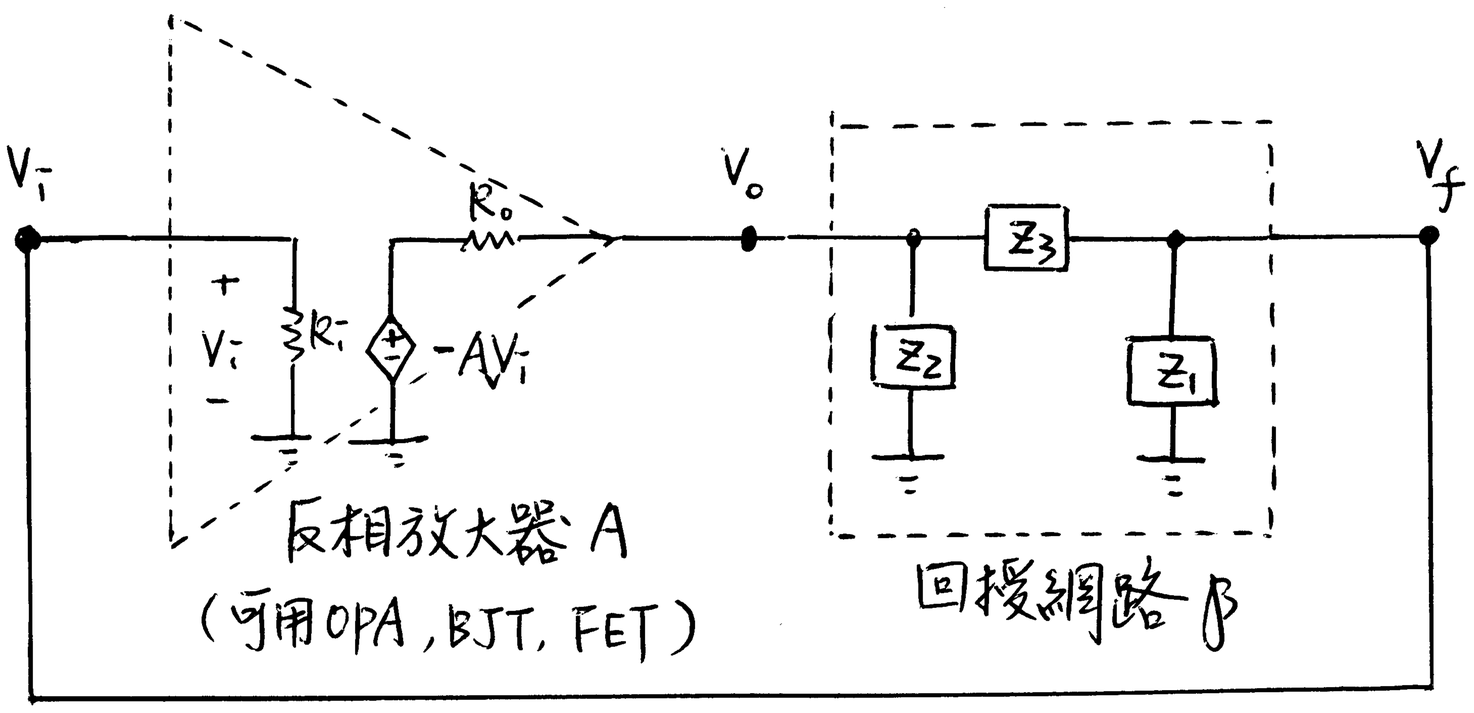
$\omega = \frac{1}{\sqrt{R_1R_2C_1C_2}}$ 為振盪頻率　　$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$

振盪條件：$|\beta A| \geq 1$，即 $A \geq \frac{1}{\beta}$，

$$1+\frac{R_3}{R_4} \geq \frac{R_1C_1+R_2C_2+R_2C_1}{R_2C_1} = \frac{R_1}{R_2}+\frac{C_2}{C_1}+1$$

$$\therefore \quad \frac{R_3}{R_4} \geq \frac{R_1}{R_2}+\frac{C_2}{C_1}$$

LC振盪器的基本架構



▲ 回授網路又稱為 LC 頻率選擇網路，其中 Z_1, Z_2, Z_3 可為電感或電容。

設 $Z_i = jX_i$, $\forall i = 1, 2, 3$, $\begin{cases} ① X_i > 0, \text{電感性元件} \\ ② X_i < 0, \text{電容性元件} \end{cases}$

▲ 迴路增益 $L = \beta A = \dfrac{V_f}{V_i}$

假設反相放大器的 R_i 很大，忽略負載效應，則

$$L = \frac{V_f}{V_i} = (-A_v)\frac{Z_2 \parallel (Z_3 + Z_1)}{R_o + [Z_2 \parallel (Z_3 + Z_1)]} \times \frac{Z_1}{Z_3 + Z_1}$$

將 $Z_1 = jX_1$, $Z_2 = jX_2$, $Z_3 = jX_3$ 代入上式，整理可得

$$L(j\omega) = \frac{A_v X_1 X_2}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

由巴克豪森準則可知，當上式虛部為零時才能符合振盪要求，

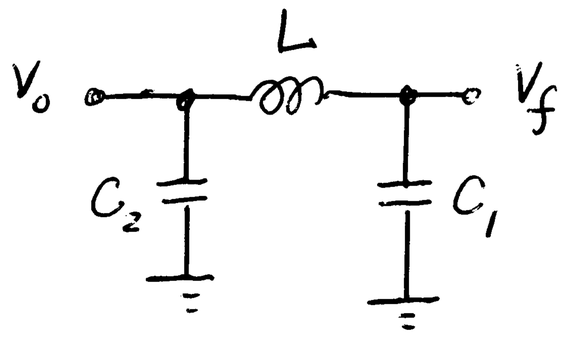
故振盪頻率 ω_o 必須滿足 $X_1 + X_2 + X_3 = 0$ 。

▲ 振盪條件

當 $X_1 + X_2 + X_3 = 0$ 時， $L = \beta A = \dfrac{A_v X_1 X_2}{-X_2(X_1 + X_3)} = A_v \dfrac{X_1}{X_2}$

因此 $|L| = |\beta A| = \left| A_v \dfrac{X_1}{X_2} \right| \geq 1$ 為振盪條件。

▲ Colpitts 振盪器



$$Z_1 = -jX_{C_1} = -j\frac{1}{\omega C_1}$$

$$Z_2 = -jX_{C_2} = -j\frac{1}{\omega C_2}$$

$$Z_3 = jX_L = j\omega L$$

1. 振盪頻率必須滿足 $X_1 + X_2 + X_3 = 0$ ，即

$$\left(-\frac{1}{\omega C_1}\right) + \left(-\frac{1}{\omega C_2}\right) + \omega L = 0$$

$$\Rightarrow \quad \omega L = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{\frac{1}{C_1} + \frac{1}{C_2}}{L}} = \sqrt{\frac{1}{LC_S}} = \frac{1}{\sqrt{LC_S}}$$

其中 $C_S = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \dfrac{C_1 C_2}{C_1 + C_2}$ ，相當於 C_1 , C_2 電容串聯。

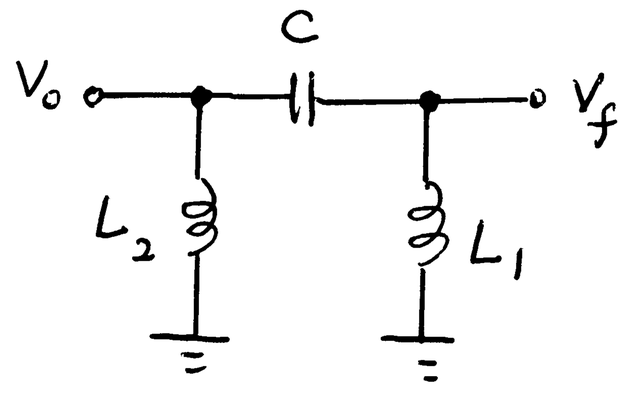
2. 振盪條件必須滿足 $\left|A_v \dfrac{X_1}{X_2}\right| \geq 1$ ，即

$$\left|A_v\left(-\frac{1}{\omega C_1}\right)(-\omega C_2)\right| \geq 1$$

$$\Rightarrow \quad \left|A_v \frac{C_2}{C_1}\right| \geq 1$$

$$\Rightarrow \quad |A_v| \geq \frac{C_1}{C_2}$$

▲ Hartley 振盪器



$$Z_1 = jX_{L1} = j\omega L_1$$

$$Z_2 = jX_{L2} = j\omega L_2$$

$$Z_3 = -jX_C = -j\frac{1}{\omega C}$$

1. 振盪頻率，必須滿足 $X_1 + X_2 + X_3 = 0$，即

$$\omega L_1 + \omega L_2 + \left(-\frac{1}{\omega C}\right) = 0$$

$$\Rightarrow \quad \omega(L_1 + L_2) = \frac{1}{\omega C}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{1}{(L_1+L_2)C}} = \sqrt{\frac{1}{L_S C}} = \frac{1}{\sqrt{L_S C}}$$

其中 $L_S = L_1 + L_2$，相當於 L_1、L_2 電感串聯。

2. 振盪條件，必須滿足 $\left|A_v \frac{X_1}{X_2}\right| \geq 1$，即

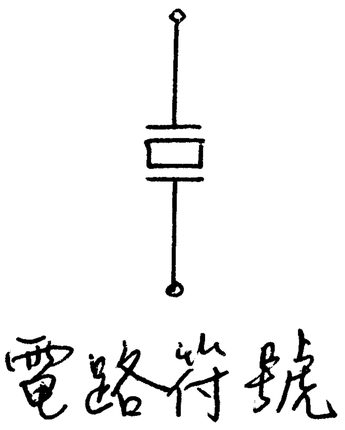
$$\left|A_v \frac{\omega L_1}{\omega L_2}\right| \geq 1$$

$$\Rightarrow \quad \left|A_v \frac{L_1}{L_2}\right| \geq 1$$

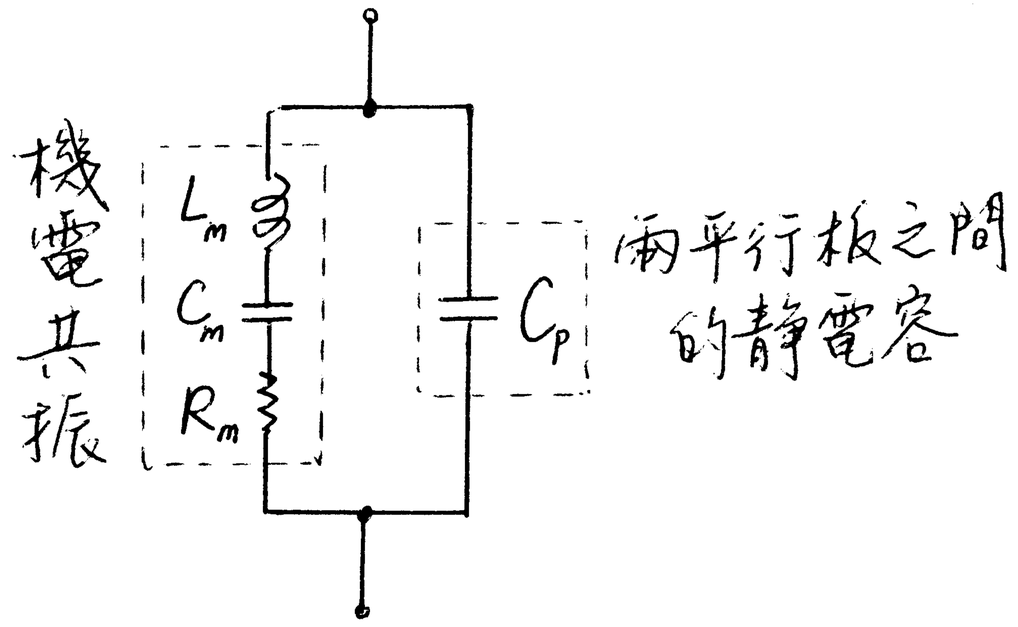
$$\Rightarrow \quad |A_v| \geq \frac{L_2}{L_1}$$

▲ 石英晶体振盪器

石英是一種压電晶体，在外加電压下會產生機電共振，對時間和溫度而言穩定性極佳，且電路對頻率的選擇性很好（Q值很高）。

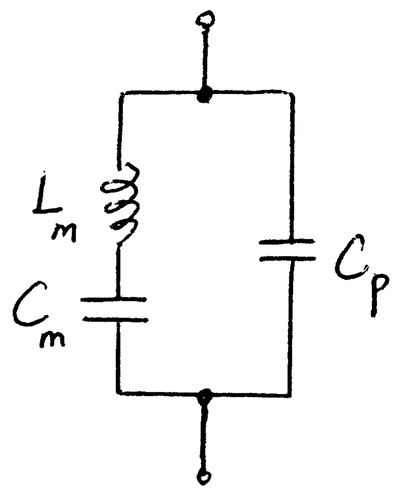


電路符號



等效電路

等效電路中 L_m 很大，R_m 很小，且 $C_p \gg C_m$，設忽略 R_m 則可簡化成



晶体的阻抗為

$$Z(s)=\frac{\left(sL_m+\frac{1}{sC_m}\right)\frac{1}{sC_p}}{\left(sL_m+\frac{1}{sC_m}\right)+\frac{1}{sC_p}}=\frac{\frac{L_m}{C_p}+\frac{1}{s^2C_mC_p}}{sL_m+\frac{1}{s}\left(\frac{1}{C_m}+\frac{1}{C_p}\right)}$$

其中 $s=j\omega$.

(1) 串联共振時，$Z(s)=Z(j\omega_s)=0$，即 $Z(s)$ 的分子為 0.

$$\frac{L_m}{C_p}+\frac{1}{-\omega_s^2C_mC_p}=0 \quad\Rightarrow\quad \frac{1}{C_p}\left(L_m-\frac{1}{\omega_s^2C_m}\right)=0$$

$\because C_p\neq 0 \quad \therefore L_m=\frac{1}{\omega_s^2C_m}$，$\boxed{\omega_s=\frac{1}{\sqrt{L_mC_m}}}$（串联共振頻率）

(2) 並联共振時，$Z(s)=Z(j\omega_p)=\infty$，即 $Z(s)$ 的分母為 0.

$$sL_m+\frac{1}{s}\left(\frac{1}{C_m}+\frac{1}{C_p}\right)=0 \quad\Rightarrow\quad s^2L_m+\left(\frac{1}{C_m}+\frac{1}{C_p}\right)=0$$

$$\Rightarrow\quad -\omega_p^2L_m+\left(\frac{1}{C_m}+\frac{1}{C_p}\right)=0 \quad\Rightarrow\quad \omega_p^2=\frac{\frac{1}{C_m}+\frac{1}{C_p}}{L_m}=\frac{1}{L_m\cdot\frac{1}{\frac{1}{C_m}+\frac{1}{C_p}}}$$

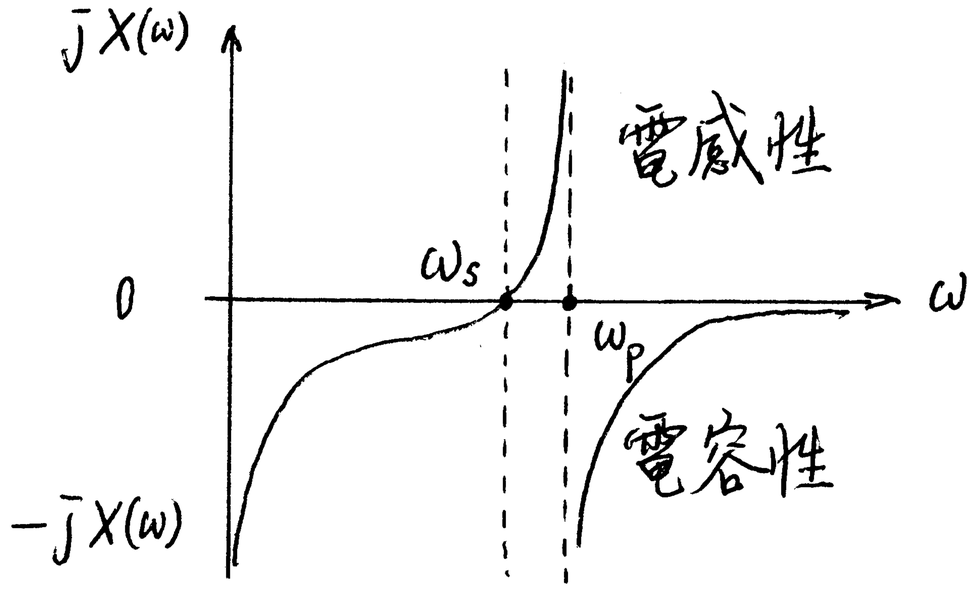
$$\Rightarrow\quad \boxed{\omega_p=\frac{1}{\sqrt{L_mC_{eq}}}}\ \text{（並联共振頻率）}$$

其中 $C_{eq}=\frac{1}{\frac{1}{C_m}+\frac{1}{C_p}}=\frac{C_mC_p}{C_m+C_p}$

(3) 理論上 $\omega_p > \omega_s$，但因為 $C_p \gg C_m$，所以 $\omega_p \simeq \omega_s$
兩個並振頻率非常接近。

(4) 晶体的阻抗可經由推導得到 $Z(j\omega) = j\,\underline{\frac{-1}{\omega C_p}\,\frac{\omega^2-\omega_s^2}{\omega^2-\omega_p^2}} = j\,X(\omega)$

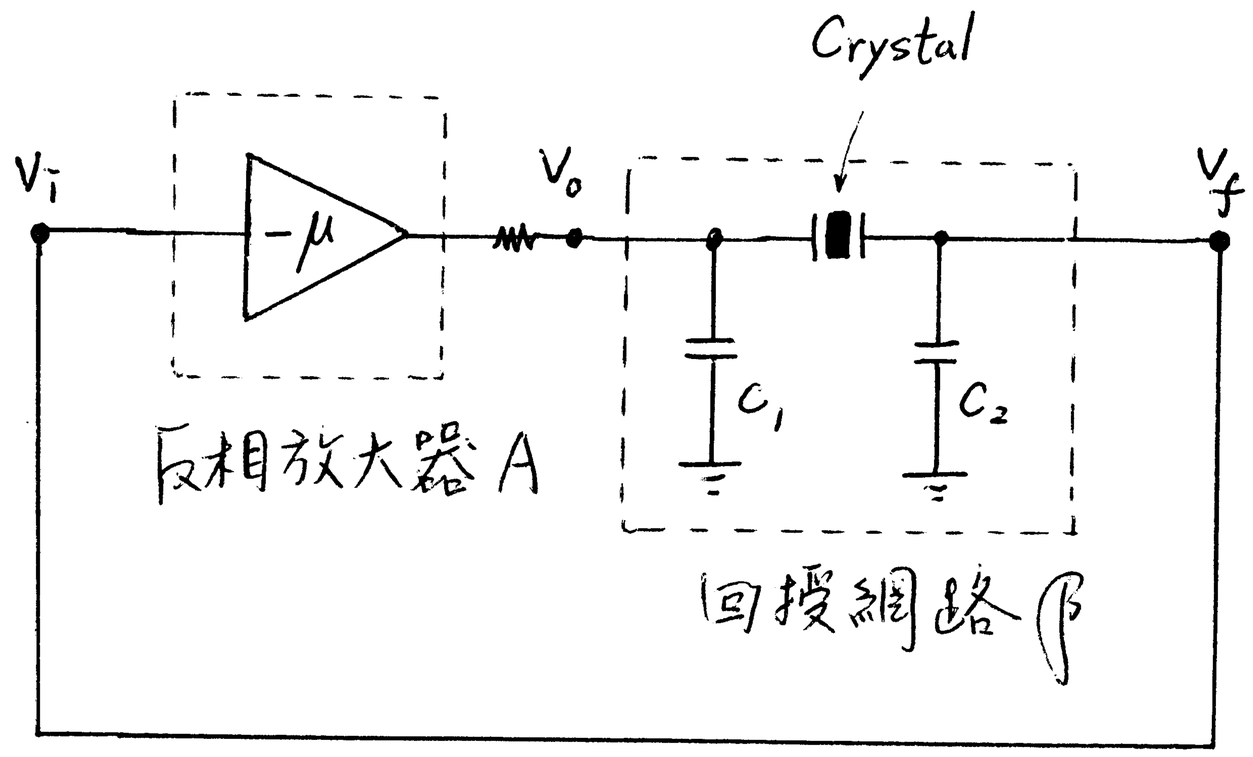
晶体電抗与頻率的関係圖如下：



晶体在 ω_s 和 ω_p 這段非常窄的頻帶之間將呈現電感性。

(5) 對於一個給定的晶体而言，上述的頻帶即已經被決定，因此可使用此晶体取代電感器作振盪器的應用。

例：Colpitts振盪器



當振盪發生時必須在頻率 ω_0 滿足 $X_1 + X_2 + X_3 = 0$ 的條件，

即 $\omega_0 = \frac{1}{\sqrt{LC_s}}$，其中 $C_s = \frac{C_1 C_2}{C_1 + C_2}$。

∵ 石英晶体在 ω_s 和 ω_p 之間才可視為電感器 ∴ $\omega_s < \omega_0 < \omega_p$

但 $\omega_s \approx \omega_p$，所以 $\omega_0 \cong \omega_s \cong \omega_p$，振盪頻率直接由石英晶体決定。